

# Reading seminar on modular sheaves on HK 2023 Spring

## Goal of the seminar

Understand the hyperholomorphic sheaves on HK introduced by Verbitsky [14] and their moduli spaces.

### 0.1 Talk 1: Introduction to hyperholomorphic sheaves

We may follow [14] [15] [16].

### 0.2 Talk 2: Markman's work on hyperholomorphic (twisted) sheaves

[9] [4][Section 5]

### 0.3 Talk 3: O'Grady's modular sheaves

[11] [12]

### 0.4 Talk 4: The LLV algebra and derived category of HK

[8] [13] [2]

### 0.5 Talk 5: Markman's modular sheaves

[10]

### 0.6 Talk 6: Beckmann's work on deformation of sheaves

[1]

### 0.7 Talk 7: Enhanced mirror symmetry

[6] [7]

### 0.8 Talk 8: Moduli of modular sheaves, what we have known ?

[5] [3]

## References

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